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Combined conduction and radiation heat transfer with variable thermal conductivity and variable refractive index

Subhash C. Mishra *, N.A. Krishna, N. Gupta, G.R. Chaitanya

Department of Mechanical Engineering, IIT Guwahati, Guwahati 781039, India

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Abstract

This article deals with the solution of conduction–radiation heat transfer problem involving variable thermal conductivity and variable refractive index. The discrete transfer method has been used for the determination of radiative information for the energy equation that has been solved using the lattice Boltzmann method. Radiatively, medium is absorbing, emitting and scattering. To validate the formulation, transient conduction and radiation heat transfer in a planar participating medium has been considered. For constant thermal conductivity and constant and variable refractive indices, results have been compared with those available in the literature. Effects of conduction–radiation parameter and scattering albedo on temperature have been studied for variable thermal conductivity and constant and/or variable refractive index. Lattice Boltzmann method and the discrete transfer method have been found to successfully deal with the complexities introduced due to variable thermal conductivity and variable refractive index.
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1. Introduction

Many heat transfer problems involve consideration of conduction, convection and/or radiation. In such cases, treatment of one mode without considering effects of the other often lead to erroneous results. Therefore, in combined mode problems, it is necessary to consider the relative importance of one mode over the other. However, with such considerations, mathematical complexity increases.

The cases of constant thermal conductivity and unity refractive index in combined conduction–radiation heat transfer problems have been studied in detail by many investigators [1,2]. Because of the mathematical complexities, a limited literature is available that individually deal with the effects of variable thermal conductivity [3,4] and constant and/or variable refractive index [5–9]. However, no work has been reported so far that deals with the combined effects of the two in a combined mode conduction–radiation heat transfer problem in a participating medium. The case of variable thermal conductivity and variable refractive index finds application in the thermal analysis of graded index medium. The present work is, therefore, aimed at the analysis of conduction and radiation heat transfer in a participating medium considering the effect of variable thermal conductivity and constant and/or variable refractive index.

The usage of the lattice Boltzmann method (LBM) to solve energy equations of heat transfer problems is gaining momentum [4,10–18] and its recent application to problems involving conduction, convection and/or radiation heat transfer has been encouraging [4,15–18]. Quite recently Gupta et al. [4] used the concept of a variable relaxation time in the LBM and solved the energy equation of a temperature dependent transient conduction and radiation heat transfer in a planar medium. Refractive index was considered unity in their study. The discrete transfer method (DTM) [19] is one of the popular methods to compute the radiative information required for the energy equation of a combined radiation, conduction and/or

* Corresponding author. Tel.: +91 361 2582660; fax: +91 361 2690762.
E-mail address: scm_iitg@yahoo.com (S.C. Mishra).

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Nomenclature

c_p	specific heat
e_i	propagation speed in the direction i in the lattice
$\vec{e}_i$	propagation velocity in the direction i in the lattice
f_i	particle distribution function in the i direction
$f_i^{(0)}$	equilibrium particle distribution function in the i direction
G	incident radiation
I	intensity
k	thermal conductivity
M_δ	total number of discrete divisions of polar angle
m	index for direction
N	conduction–radiation parameter
n	refractive index
p	anisotropy factor
q_R	radiative heat flux
S	source term
s	geometric distance
T	temperature
t	time
X	thickness of the medium
x	coordinate direction

Greek symbols

α	thermal diffusivity
β	extinction coefficient
γ	coefficient for thermal conductivity variation
γ'	variable thermal conductivity parameter
ε	emissivity
θ	non-dimensional temperature
ξ	non-dimensional time
ρ	density
σ	Stefan–Boltzmann constant
σ_s	scattering coefficient
Ψ	non-dimensional heat flux
τ	relaxation time
ω	scattering albedo

Subscripts

av	average
b	boundary
d,u	downstream, upstream
0	reference
E,W	east, west
R	radiative

convection mode problem. Its application to a pure radiative transfer problem dealing with variable refractive index participating medium has been recently extended by Krishna and Mishra [9].

One another objective of the present article is, therefore, also to extend the concepts of the variable relaxation time in the LBM proposed by Gupta et al. [4] and that of the variable refractive index in the DTM by Krishna and Mishra [9] to analyze the combined effects of the two in a conduction–radiation heat transfer problem.

In the following pages, we first provide a general formulation of the DTM and the LBM to analyze conduction–radiation heat transfer with variable thermal conductivity and variable refractive index. Next, to validate the LBM-DTM formulations, some representative results are compared with those available in the literature [5,6] for constant thermal conductivity and constant non-unity and/or variable refractive index. Results for variable thermal conductivity and constant and/or variable refractive index are presented next. These results are presented for the effects of the conduction–radiation parameter and scattering albedo on temperature distributions. Conclusions are made at the end.

2. Formulation

Consider a 1-D planar conducting–radiating medium (Fig. 1) with variable thermal conductivity k and variable refractive index n. Other thermophysical properties such as density ρ, specific heat c_p, and optical properties such as extinction coefficient β and scattering albedo ω are assumed constant. The system is initially at temperature T_E and for time $t > 0$, its west boundary is raised to temperature T_W. The variation of thermal conductivity with temperature is taken as

$$k = k_0 + \gamma'(T - T_W) \tag{1}$$

where k_0 is the reference thermal conductivity and γ' is the coefficient of thermal conductivity variation. The refractive index n of the medium assumes either a constant non-unity value ($n > 1$) or varies linearly with distance $n_x = n_W + \left(\frac{n_E - n_W}{L}\right)x$, where n_W and n_E are the refractive indices on the west and the east faces of the medium, respectively.

For the problem under consideration, the energy equation is given by

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) - \frac{\partial q_R}{\partial x} \tag{2}$$

where q_R is the radiative heat flux and $\frac{\partial q_R}{\partial x}$ for a medium with a variable refractive index is given by [5]

$$\frac{\partial q_R}{\partial x} = \kappa_a\left[4\pi n^2 \frac{\sigma T^4}{\pi} - G\right] \tag{3}$$

where κ_a is the absorption coefficient and G is the incident radiation.

To solve for G at any location x, information about the intensity I distribution is required which for any direction s is obtained from the following radiative transfer equation [9].

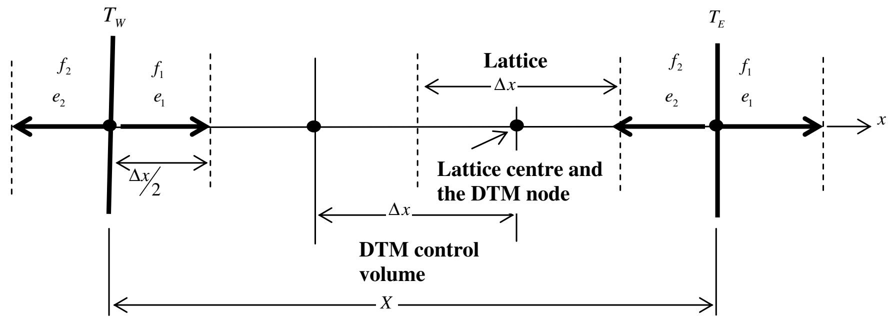

Fig. 1. One-dimensional planar geometry under consideration. Lattices of the LBM and the control volumes of the DTM are staggered.

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{I}{n^2}\right)+\beta\frac{I}{n^2}=\frac{S}{n^2} \tag{4}$$

where the source term S in terms of G and the net radiative heat flux q_R for a linear anisotropic phase function ($p = 1 + a\cos\delta\cos\delta'$) is given by

$$S=(\kappa_a n^2)\frac{\sigma T^4}{\pi}+\frac{\sigma_s}{4\pi}(G+a\cos\delta q_R) \tag{5}$$

where σ_s is the scattering coefficient, p is the scattering phase function and δ is the polar angle. Between the upstream point u and the downstream point d, if the optical path-leg $\beta\times\Delta s$ is small enough, the source term $\frac{S}{n^2}$ in Eq. (4) can be assumed constant over the path-leg and Eq. (4) after integration can be written in the recursive form as

$$\frac{I_d}{n_d^2}=\frac{I_u}{n_u^2}\exp(-\beta\times\Delta s)+S_{av}[1-\exp(-\beta\times\Delta s)] \tag{6}$$

where n_u and n_d are the refractive indices at the upstream point and the downstream point, respectively and S_{av} is the constant value of the source term over the optical path-leg between the two points. In a planar medium, S_{av} is taken as

$$S_{av}=\frac{1}{2}\left(\frac{S_d}{n_d^2}+\frac{S_u}{n_u^2}\right) \tag{7}$$

In the application of the DTM to a variable refractive index participating medium, rays take curved trajectories and for certain directions, internal reflections also occur. Over these curved trajectories, Eq. (6) is used recursively. Details of ray tracing procedure in the DTM have been given in Krishna and Mishra [9].

For a planar medium, incident radiation G is given by and in the DTM, it is numerically computed as [19]

$$G=2\pi\int_{\delta=0}^{\pi}I(\delta)\sin\delta\,\mathrm{d}\delta \approx 4\pi\sum_{l=1}^{M_\delta}I(\delta_l)\sin\delta_l\sin\left(\frac{\Delta\delta_l}{2}\right) \tag{8}$$

where M_δ is the number of discrete divisions considered over the complete span of the polar angle δ ($0\leqslant\delta\leqslant\pi$). The net radiative heat flux q_R in the DTM is numerically computed as [19]

$$q_R=2\pi\int_{\delta}^{\pi}I(\delta)\cos\delta\sin\delta\,\mathrm{d}\delta \approx 2\pi\sum_{l=1}^{M_\delta}I(\delta_l)\cos\delta_l\sin\delta_l\sin\Delta\delta_l \tag{9}$$

In the DTM, intensities are traced from the boundaries. In Eq. (6), for a given direction δ_l, if the upstream point lies on the boundary, then its values have to be computed from the radiative boundary condition. For a diffuse-gray boundary with temperature T_b and emissivity ε_b, the boundary intensity I_b is given by

$$I_b=(n^2\varepsilon_b)\frac{\sigma T_b^4}{\pi}+\frac{(1-\varepsilon_b)}{\pi}2\pi\sum_{l=1}^{M_\delta/2}I(\delta_l)\cos\delta_l\sin\delta_l\sin\Delta\delta_l \tag{10}$$

Once the radiative information $\partial q_R/\partial x$ is known from Eq. (3), the next step is to introduce the effect of the variable thermal conductivity in the energy equation (Eq. (2)).

Substituting for k from Eq. (1) into Eq. (2), we get

$$\rho c_p\frac{\partial T}{\partial t}=[k_0+\gamma'(T-T_W)]\frac{\partial^2 T}{\partial x^2}+\gamma'\left(\frac{\partial T}{\partial x}\right)^2-\frac{\partial q_R}{\partial x} \tag{11}$$

With non-dimensional temperature θ, distance x^*, conduction–radiation parameter N, radiative heat flux Ψ_R, time ξ and variable thermal conductivity parameter γ defined in the following way:

$$\theta=\frac{T}{T_W},\quad x^*=\beta x,\quad N=\frac{k_0\beta}{4\sigma T_W^3},\quad \Psi_R=\frac{q_R}{\sigma T_W^4},\quad \xi=\alpha\beta^2 t,\quad \gamma=\frac{\gamma' T_W N}{k_0} \tag{12}$$

Eq. (11) becomes

$$\frac{\partial\theta}{\partial\xi}=\left(1+\frac{\gamma(\theta-1)}{N}\right)\frac{\partial^2\theta}{\partial x^{*2}}+\frac{\gamma}{N}\left(\frac{\partial\theta}{\partial x^*}\right)^2-\frac{1}{4N}\frac{\partial\Psi_R}{\partial x^*} \tag{13}$$

The initial and the boundary conditions for the problem under consideration are as follows:

$$\begin{aligned}&\text{Initial condition}: \theta(x^*,0)=\theta_E\\&\text{Boundary conditions}: \theta(0,\xi)=\theta_W,\quad \theta(X^*,\xi)=\theta_E\end{aligned} \tag{14}$$

Eq. (13) is a non-linear partial differential equation in which apart from the non-linearity in the radiative term $\partial\Psi_R/\partial x^*$, the first two terms on the right-hand side are also non-linear. For removing the non-linearity due to radiative term, the radiative information is calculated from the temperature values from the previous iteration. However, non-linearity in other terms of the energy equation requires separate treatment whose solution in the conventional CFD methods is given in Talukdar and Mishra [3]. The treatment of non-linearity in the LBM is different and the same has been explained in Gupta et al. [4].

In the LBM, the discrete Boltzmann equation with Bhatanagar–Gross–Krook (BGK) approximation is given by [11]

$$\frac{\partial f_i(\vec{x},t)}{\partial t} + \vec{e}_i \cdot \nabla f_i(\vec{x},t) = -\frac{1}{\tau}[f_i(\vec{x},t) - f_i^{(0)}(\vec{x},t)] \tag{15}$$

where f_i is the particle distribution function denoting the number of particles at the lattice node $\vec{x}$ and time t moving in direction i with velocity $\vec{e}_i$ along the lattice link $\Delta x = e_i\Delta t$ connecting the neighbors, τ is the relaxation time and $f_i^{(0)}$ is the equilibrium distribution function. For the 1-D planar medium problem under consideration (Fig. 1), τ for the D1Q2 lattice is computed from [11]

$$\tau = \frac{\alpha}{|\vec{e}_i|^2} + \frac{\Delta t}{2} \tag{16}$$

where α is the thermal diffusivity. The two velocities e_1 and e_2, and their corresponding weights w_1 and w_2 for the D1Q2 lattice are given by

$$e_1 = \Delta x/\Delta t, \quad e_2 = -\Delta x/\Delta t, \quad w_1 = w_2 = \frac{1}{2} \tag{17}$$

After discretization, Eq. (15) is written as

$$f_i(\vec{x} + \vec{e}_i\Delta t, t+\Delta t) = f_i(\vec{x},t) - \frac{\Delta t}{\tau}[f_i(\vec{x},t) - f_i^{(0)}(\vec{x},t)] \tag{18}$$

Eq. (18) is the LB equation with BGK approximation that describes the evolution of the particle distribution function f_i.

In a problem involving conduction and radiation, temperature is obtained after summing f_i over all directions [11], i.e.

$$T(\vec{x},t) = \sum_i f_i(\vec{x},t) \tag{19}$$

To process Eq. (18), an equilibrium distribution function is required. For the problem under consideration, this is given by

$$f_i^{(0)}(\vec{x},t) = w_i T(\vec{x},t) \tag{20}$$

From Eqs. (19) and (20), we have

$$\sum_i f_i^{(0)}(\vec{x},t) = \sum_i w_i T(\vec{x},t) = T(\vec{x},t) = \sum_i f_i(\vec{x},t) \tag{21}$$

Eq. (18) provides the solution of a transient heat conduction problem in the LBM. To account for the radiation term in the energy equation (Eq. (13)), in the LBM formulation, Eq. (18) is modified to [15,16]

$$f_i(\vec{x} + \vec{e}_i\Delta t, t+\Delta t) = f_i(\vec{x},t) - \frac{\Delta t}{\tau}\left[f_i(\vec{x},t) - f_i^{(0)}(\vec{x},t)\right] - \frac{\Delta t w_i}{\rho c_p}\frac{\partial q_R}{\partial x} \tag{22}$$

Eq. (22) is the equivalent form of the energy equation (Eq. (11)) in the LBM formulation. To account for the temperature dependent thermal conductivity (Eq. (1)), the expression for the relaxation time τ in Eq. (16) becomes

$$\tau = \frac{k/\rho c_p}{|\vec{e}_i|^2} + \frac{\Delta t}{2} = \frac{k_0 + \gamma'(T - T_W)}{\rho c_p|\vec{e}_i|^2} + \frac{\Delta t}{2} = \frac{k_0}{\rho c_p|\vec{e}_i|^2} + \frac{\gamma'}{\rho c_p|\vec{e}_i|^2}(T - T_W) + \frac{\Delta t}{2} \tag{23}$$

With non-dimensional quantities as defined in Eqs. (12), (22) in non-dimensional form is given by

$$f_i^*(\vec{x}^* + \vec{e}_i^*\Delta\xi, \xi + \Delta\xi) = f_i^*(\vec{x}^*,\xi) - \frac{\Delta\xi}{\tau^*}\left[f_i^*(\vec{x}^*,\xi) - f_i^{*(0)}(\vec{x}^*,\xi)\right] - \frac{\Delta\xi w_i}{4N}\frac{\partial\Psi_R}{\partial x^*} \tag{24}$$

where f_i^* is the particle distribution function in non-dimensional from and it is computed from the non-dimensional temperature. In Eq. (24), $\frac{\partial\Psi_R}{\partial x^*}$ is given by

$$\frac{\partial\Psi_R}{\partial x^*} = 4(1-\omega)\left(n^2\theta^4 - \frac{G^*}{4\pi}\right) \tag{25}$$

where $G^* = G\Big/\left(\frac{\sigma T_W^4}{\pi}\right)$. Eq. (23) for the relaxation time takes the following form

$$\tau^* = \frac{1}{(\Delta x^*/\Delta\xi)^2} + \frac{\gamma}{(\Delta x^*/\Delta\xi)^2}(\theta - \theta_W) + \frac{\Delta\xi}{2} \tag{26}$$

With radiative information known from any of the methods like the DTM, the DOM and the FVM, the procedure to solve energy equation of a conduction–radiation problem using the LBM has been given in [15–18].

3. Results and discussion

To validate the LBM and the DTM formulations for the treatment of variable thermal conductivity and variable refractive index presented above, we consider a 1-D planar medium of 10 cm thickness. Its west boundary temperature $T_W > T_E$. The refractive index of the medium can either have a non-unity ($n > 1$) constant value or a linear variation $n_x = n_W + \left(\frac{n_E - n_W}{L}\right)x$. The thermal conductivity has a linear dependence on temperature with its variation as given in Eq. (1). Thermal conductivity parameter γ can take positive, negative or zero values.

For grid-independent solution, the medium was divided into 100 lattices/control volumes for constant refractive index case and that for a variable refractive index case, it was divided into 1000 lattices/control volumes. For ray-independent solution, 36 equally-spaced directions were considered. In the following pages, results have been presented for steady-state condition. However, the problem was solved as a transient one. The steady-state condition was assumed to have reached when temperature difference between two consecutive time levels at each lattice center did not exceed 1×10^{-6}. In solving the energy equation, non-dimensional time step $\Delta\xi = 0.0001$ was considered.

In Figs. 2 and 3, results of the present work have been validated against those available in the literature [5,6]. Comparison of the non-dimensional temperature θ results for constant refractive index $n_W = n_E = 1.5$ and constant thermal conductivity ($\gamma = 0.0$) has been made in Fig. 2. For absorbing-emitting $\omega = 0.0$ and extinction coefficient $\beta = 10.0$, these comparisons have been given for the conduction–radiation parameter $N = 0.1306$ and 0.01306. Comparisons of the present results for the two values of $N = \frac{k_0\beta}{4\sigma T_W^3}$ given in Fig. 2 are based on the dimensional values of k_0 and β taken from Abdallah and Dez [5] where $T_E = 1000$ K and $T_W = 1500$ K. It is seen from the Fig. 2 that the results of the present work match very well with that of Abdallah and Dez [5] which are based on the curved ray tracing approach for radiation and the finite difference scheme for the energy equation.

In the case of a linearly varying refractive index ($n_W = 1.2$, $n_E = 1.8$) medium with constant thermal conductivity ($\gamma = 0.0$), temperature results of the present work have been compared with that of Xia et al. [6] in Fig. 3. This comparison has been shown for $\beta = 10.0$, $\omega = 0.0$ and $N = 0.01306$. Comparisons of the present results in Fig. 2 for $N = \frac{k_0\beta}{4\sigma T_W^3} = 0.01306$ is based on $k_0 = 0.1$ Wm^{-1} K^{-1} and $T_W = 1500$ K given in [6]. T_E in Xia et al. [6] was taken as 1000 K. It is seen from Fig. 3 that present results match very well with that of [6].

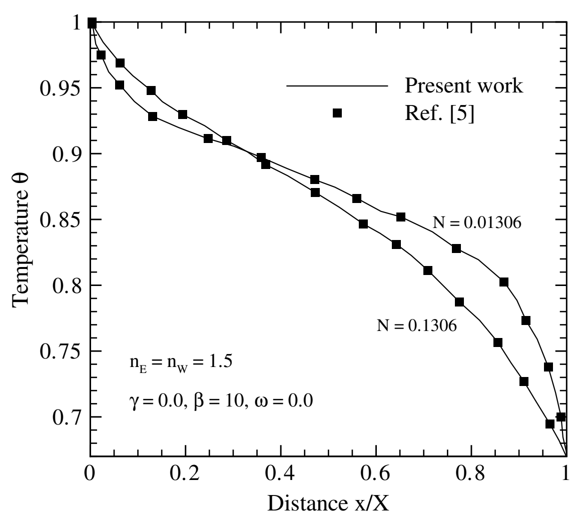

Fig. 2. Comparisons of non-dimensional temperature θ variation with distance x/X of the present work with that of Abdallah and Dez [5].

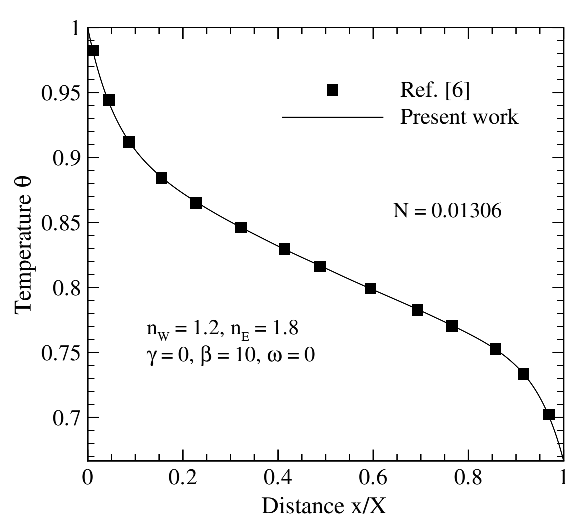

Fig. 3. Comparisons of non-dimensional temperature θ variation with distance x/X of the present work with that of Xia et al. [6] for $N = 0.01306$.

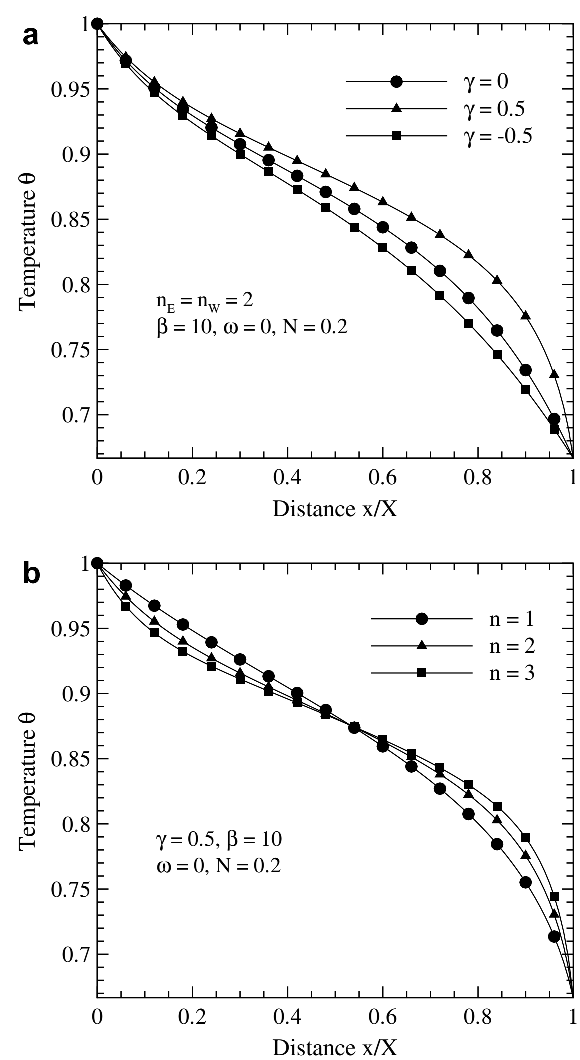

Fig. 4. Non-dimensional temperature θ distribution with distance x/X (a) effect of γ for $n = n_W = n_E$ and (b) effect of n for γ.

For constant thermal conductivity, having validated the general formulations of the LBM and the DTM for non-unity constant refractive index with Abdallah and Dez [5] and for variable refractive index with Xia et al. [6], in the following pages, we provide some results to show the effects of variable thermal conductivity and constant and/or variable refractive index. These results are shown for various radiative parameters.

In Fig. 4a, effect of variable thermal conductivity with $\gamma = -0.5$, 0.0 and 0.5 on θ variation in the medium has been shown for refractive $n_W = n_E = 2.0$. In Fig. 4b, effect of refractive index $n = n_W = n_E$ on θ has been shown. Results in Fig. 4a and b are for $\beta = 10.0$, $\omega = 0.0$ and $N = 0.2$. It is seen from Fig. 4 that for a constant refractive index n, variable thermal conductivity parameter γ is having significant effect on θ, and for a given γ, n is also having a major effect on θ.

It is seen from Fig. 4a that near the cold boundary, the effect of γ is more. Compared to $\gamma = -0.5$, at any location, with respect to $\gamma = 0.0$, temperature θ difference and non-linearity are more for $\gamma = 0.5$. That is attributed to the fact that for $\gamma = 0.5$, thermal conductivity increases with increases in temperature, and radiation being a high temperature phenomenon, it becomes more prominent. From Fig. 4b, it is observed that with increase in refractive index n, θ near the hot boundary decreases. Near the cold boundary, an opposite trend is observed. θ profile is more non-linear for higher values of n. This trend is owing to the fact that the refractive index n also contributes to the non-linearity. It appears as n^2 term in the divergence of radiative heat flux term (Eq. (25)) that is the radiation source term in the energy equation (Eq. (22)).

In Fig. 5a and b, effects of the conduction–radiation parameter N and scattering albedo ω have been shown on θ. These effects have been shown for $n = n_W = n_E = 2.0$, $\gamma = 0.5$ and $\beta = 10.0$. For results in Fig. 5a, $\omega = 0.0$ and effects have been studied for the conduction–radiation parameter $N = 0.2$, 0.35 and 0.5. It is seen that initially

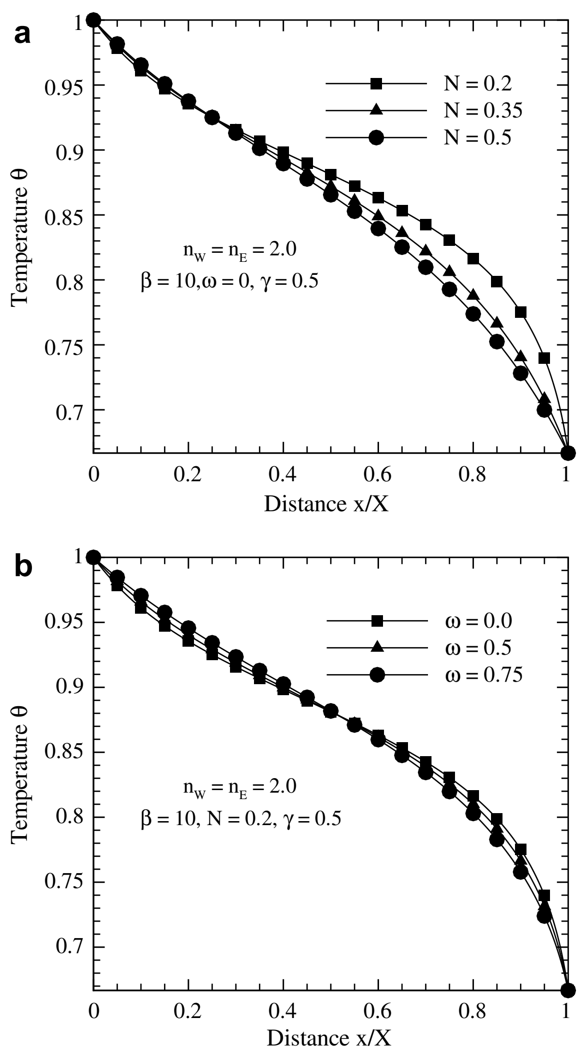

Fig. 5. Non-dimensional temperature θ distribution with distance x/X (a) effect of N for $n = n_W = n_E$ and (b) effect of ω for $n = n_W = n_E$.

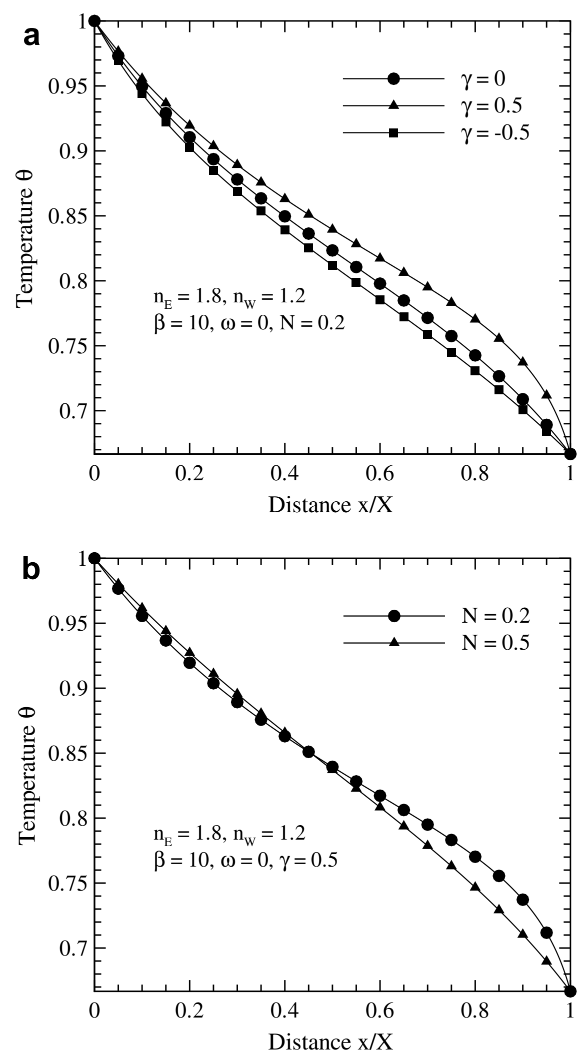

Fig. 6. For the case of variable refractive index $n_E = 1.8$, $n_W = 1.2$ (a) effect of γ on non-dimensional temperature θ variation with distance x/X and (b) effect of N on non-dimensional temperature θ variation with distance x/X.

the temperature drops for all three values of N are similar. However, for the radiation dominated case ($N = 0.2$) towards the cold boundary, the temperature drop is sharp. In Fig. 5b, for the case of $N = 0.2$, θ results are presented for $\omega = 0.0$, 0.5 and 0.75. The θ profiles for all the three cases are found to intersect at the mid-plane. For a lower value of ω, temperature change is more towards the boundaries.

Effect of γ on θ distribution for the case of a linearly varying refractive index $n_W = 1.2$, $n_E = 1.8$ has been shown in Fig. 6a. In Fig. 6b, the effect of the conduction–radiation parameter N has been shown.

Comparison of results in Figs. 4a and 6a shows that the effect of γ on θ is more towards the cold (east) boundary. The different values of $n = n_W = n_E$ have no effect on θ at the mid-plane (Fig. 4b). However, effect increases away from the mid-plane. Towards the hot (west) boundary, at any location, θ decreases with increase in the refractive index n. An opposite trend is observed towards the cold (east) boundary. The effect on N is found more towards the cold boundary (Fig. 6b).

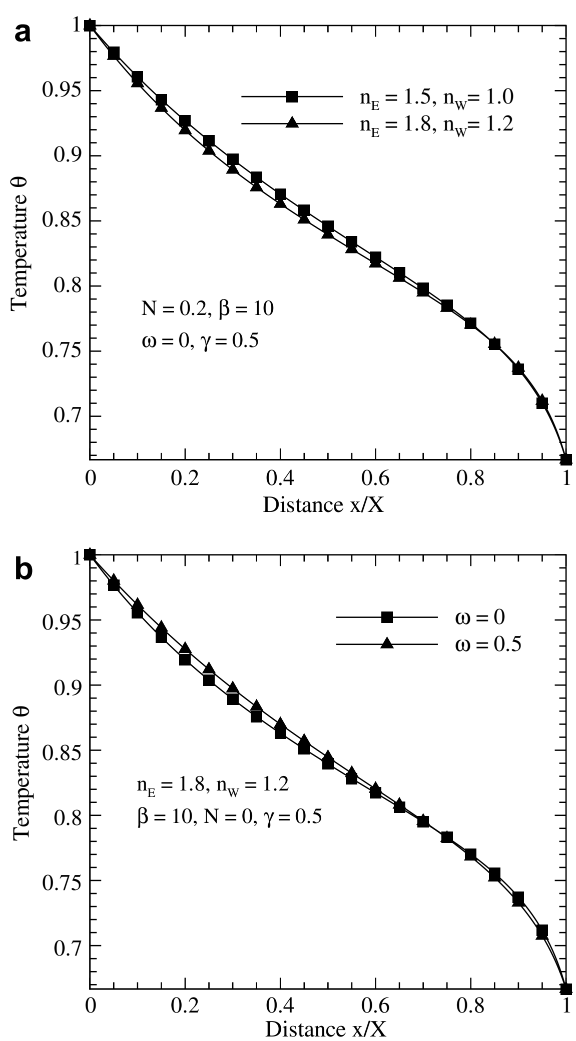

Fig. 7. For the case of variable refractive index (a) non-dimensional temperature θ variation with distance x/X for $n_E = 1.8$, $n_W = 1.2$ and $n_E = 1.5$, $n_W = 1.0$ (b) effect of ω on non-dimensional temperature θ variation with distance x/X for $n_E = 1.8$, $n_W = 1.2$.

In Fig. 7a, effect of the variable refractive index on θ has been given for $N = 0.2$, $\beta = 10.0$, $\omega = 0.0$ and $\gamma = 0.5$. In Fig. 7b, for $n_E = 1.8$, $n_W = 1.2$, effect of ω on θ has been given for $N = 0.2$, $\beta = 10.0$ and $\gamma = 0.5$. It is seen from Fig. 7a that for $n_E = 1.8$, $n_W = 1.2$, temperature drop near the hot boundary is more than that for $n_E = 1.5$, $n_W = 1.0$. Near the cold boundary, in both the cases, temperature drops at the same rate. For the effect of ω, in case of $n_E = 1.8$, $n_W = 1.2$, it is seen that when scattering is more, the temperature changes near the hot and the cold boundaries are less.

4. Conclusions

The LBM and the DTM were used to analyze combined conduction–radiation heat transfer in a planar absorbing, emitting and scattering medium with variable thermal conductivity and constant and/or variable refractive index. The radiative information was computed using the DTM, and the energy equation was solved using the LBM. The results of the LBM-DTM formulation were compared for the constant thermal conductivity case and constant non-unity as well as variable refractive index with those available in the literature. A very good agreement was found. Effects of variable thermal conductivity and variable refractive index on temperature distributions were studied. These were found to have significant bearings on the results.

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